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ABSTRACT

This booklet is one in a series of teacher inservice materials developed in an effort to translate the findings of a study of problem-solving instruction into materials that provide teachers with new knowledge about mathematical problem solving, current instructional practices, and recommendations for problem-solving instruction. The need for problem-solving instruction is discussed first, with results of surveys of achievement and recommendations noted. Then goals for problem-solving instruction are presented, with the emphasis on problem solving as a process. How problem solving can expand the curriculum, types of problems, and good problems are then described. Next, teaching problem solving is discussed, with sections on problem-solving strategies, characteristics of good problem solvers and of good problem-solving instruction, instructional planning, the role of calculators, successful instructional techniques, and evaluation. A three-page annotated bibliography of selected problem-solving materials is included. (MNS)

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IDEAS ON TEACHING PROBLEM SOLVING IN INTERMEDIATE MATHEMATICS

A Guidebook for Teachers

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August 30, 1985

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PREFACE

This guidebook is part of a series of teacher inservice materials produced by the Secondary Science and Mathematics Improvement (SSAMI) Program at the Far West Laboratory for Educational Research and Development. During the 1983-1984 school year, one of the ongoing projects was a study of problem-solving instruction in seventh-grade classrooms, the Problem Solving in Intermediate Mathematics Study. This booklet and two others in the series represent an effort to translate the background and findings of the study into a set of materials that provide teachers with new knowledge about mathematical problem solving. current instructional practices, and recommendations for problem-solving instruction. The booklets are:

1. Arithmetic Word Problems: Activities to Engage Students in Problem Analysis

2. A Look at Math Teachers and Problem Solving: A Summary of a SSAMI Research Study

3. Ideas on Teaching Problem Solving in Intermediate Mathematics

We thank the teachers and students who took part in the study of problem solving for allowing observers into their classrooms, for answering our questions, and for sharing their ideas about problem solving. We also thank Dr. John Taylor, Teaching and Learning Division, National Institute of Education, for his support in this and other work. His interest in exploring innovative ways of approaching the problems that confront educators and their encouragement of educational excellence is appreciated.

We especially wish to acknowledge the contributions of Mr. Gary Tsruda, a junior high teacher and mathematics curriculum specialist. His knowledge about mathematics, classroom instruction, and inservice programs were invaluable in the preparation of this guidebook.

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THE NEED FOR PROBLEM-SOLVING INSTRUCTION

In the past few years, problem solving has become one of the most important topics in mathematics education. Many educators believe that instruction in mathematics has focused on development of computational abilities, perhaps ignoring development of problem-solving abilities. Surveys of student achievement support this view.

There is a pattern of results found repeatedly in surveys of mathematics achievement: Students able to solve computational items correctly are unable to apply the same computational skill to solve problem situations. For example, in 1982 the National Assessment of Educational Progress[1] tested 9-, 13- and 17-year olds across the nation. In this survey, 13-year olds were asked to compute the product of two fractions. They were also asked to solve a simple word problem which required the same computational skill. While over 60% of the students were able to select the correct answer to the computational exercise, only 17% selected the correct answer to the word problem.

Similar results were obtained in the 1983 California Assessment Program[2] and in the U.S. sample of the Second International Study of Mathematics[3]. They indicate that even students who are fairly proficient in basic computation are unable to apply their arithmetic knowledge to solve problem situations.

In order to improve students' problem-solving skills, educators have recommended changes be made to mathematics curricula. In 1980, the National Council of Teachers of Mathematics published An Agenda for Action in which they set forth recommendations for school mathematics in the decade of the 80's. Their major recommendation was that problem solving be identified as the focus of school mathematics. Many state guidelines for mathematics curricula have been altered to emphasize problem solving. For example, the Mathematics Framework of the California State Department of Education recommended that all mathematics concepts and skills be taught within the context of problem solving.

These recommendations were not made out of a sense of panic about the current state of mathematics education. Unlike the "New Math" reforms of the 1960's, they do not call for a total revision of the mathematics curriculum and the infusion of numerous new concepts and ideas. Rather, they suggest a shift in emphasis from a successful program of instruction in computational skills to one in which the application of these skills to new problem situations is highlighted. By shifting the focus of our mathematics curriculum to problem solving, we will begin to address an area of critical need for our students while at the same time providing them with exciting, challenging learning experiences.

GOALS FOR PROBLEM-SOLVING INSTRUCTION

In order to provide a successful problem-solving program, it is necessary to acknowledge the unique nature of the instructional process in this area. Most teachers are comfortable with the traditional methods of teaching arithmetic. Since most arithmetic programs concentrate on the acquisition of computational skills, these methods tend to be expository in nature, with the teacher doing most of the talking in order to "dispense" knowledge and skills to the students. Students spend class time listening to the teacher and then practicing the arithmetic procedure the teacher just explained. Daily homework consists of a set of drill exercises of the same type. This is an effective method of teaching arithmetic algorithms.

Problem solving, however, is not an algorithmic skill; it is a process of applying previously-learned concepts and skills to new situations. The first step in organizing a program to provide instruction in problem solving is to become aware of the essential distinction between problem solving and computation.

The goals of instructional programs that emphasize problem solving are very different from the goals of computational skills programs. In a totally computation-based program, all instructional activities are aimed at having the student produce the correct answer quickly. The algorithm is a method of arriving at that correct answer, and if it is followed correctly and all prerequisite skills have been mastered, success is guaranteed. In a sense, computational instruction involves little more than training students to follow the rules set forth in the algorithm.

Problem-solving instruction, on the other hand, is not nearly so clear-cut because problem solving is a process. In problem solving, we are concerned with the correct solution, but the solution consists of more than just the answer. It includes the methods used to arrive at the answer. The instructional goals are for students to think, to analyze situations, and to apply appropriate skills to arrive at solutions. There are no algorithms, no quick-fixes, nor short cuts. The right answer, if indeed there is one right answer, isn't nearly as important as the thinking and analysis which preceded it.

Clearly, the teaching of problem solving involves very different techniques from those which are effective in teaching algorithms, and making the shift to process-oriented instruction will be a difficult transition for some teachers. However, most teachers will enjoy the challenge and will find that the same general principles apply to good teaching regardless of context. Applying these principles in an instructional program which maintains problem solving as a goal will allow both students and teachers to make a smooth transition to a problem-solving curriculum.

PROBLEM SOLVING CAN EXPAND THE CURRICULUM

The average student's response to the question "What is mathematics?" would probably include some reference to numbers. We build the concept of numbers into our instruction to such an extent that students think of mathematics and numbers as synonymous. Because of this, students often don't know how to react when presented with mathematical problems that do not involve numbers. For example, how many students would think of this as a mathematics problem?

SAMPLE PROBLEM #1

A man was traveling with a fox, a chicken, and a bag of grain. Only his presence kept the fox from eating the chicken and the chicken from eating the grain. He came to a stream which he was able to ford carrying only one of his three possessions.

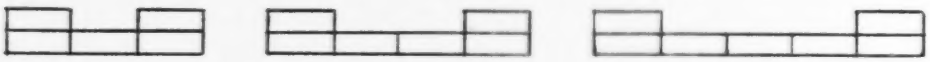
How did he get the fox, the chicken, and the bag of grain across the stream safely and uneaten?

Mathematics is the study of patterns. We use the concept of numbers to quantify and examine the relationships involved in patterns. Problem-solving instruction can help students to see structure throughout mathematics by presenting situations from all strands of the curriculum: Numbers and operations, geometry, measurement, probability and statistics, functions, and logical thinking. With problem solving, students can see the interesting challenge of math's patterns.

The following problems involve concepts from several different strands and require students to examine patterns and determine relationships among parts:

SAMPLE PROBLEM #2

If each ▭ is made up of two 1-cm squares, then find the perimeter of the next figure in the pattern.



SAMPLE PROBLEM #3

What are the next three numbers in this sequence?

1, 1, 2, 3, 5, 8, _, _, _

SAMPLE PROBLEM #4

How many dots will there be in the tenth figure of this pattern?

SAMPLE PROBLEM #5

What comes next in this sequence?

O, T, T, F, F, S, S, E, _

WHAT IS A PROBLEM?

A problem is a situation in which a person is seeking some goal for which a suitable course of action is not immediately apparent. If the problem requires the application of mathematical skills or concepts to arrive at the goal, it is a mathematical problem. Table 1 lists four types of problems a well-rounded curriculuim might include.

According to this definition, the drill exercises found in mathematics texts are not problems at all because the correct answer can be obtained by an obvious course of action, namely, the algorithm which is being practiced in doing the exercises. Students must be able to develop, execute, and evaluate a plan for solving a problem. These processes require general problem-solving approaches rather than algorithmic rules routinely applied to specific exercises.

Table 1

Types of Problems*

1. TRANSLATION PROBLEM: A story problem which can involve the use of one or more operations. Textbooks contain good ideas for use with students in this area. Translation problems provide students with experience in translating real-world situations into mathematical expressions.

   Example: Notebooks are packed 24 to a carton. If three cartons of notebooks were sold for $18.00, how much did each notebook cost?

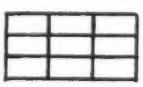
2. PROCESS PROBLEM: A problem whose solution requires the use of thinking processes such as planning, guessing, estimating, forming conjectures, and looking for patterns. Textbooks contain very few process problems. This type of problem helps students to develop general problem-solving strategies and procedures.

   Example: Seven boys and five girls were at a party. They decided to play records and dance. How many different couples (1 boy and 1 girl) could be formed?

3. APPLIED PROBLEM: A realistic problem whose solution requires the use of mathematics. An applied problem usually involves much more than the use of mathematics, but mathematics plays a key role in its solution. Applied problems make students aware of the role mathematics plays in everyday problem-solving situations.

   Example: Which letters occur most frequently in the English language?

4. PUZZLE PROBLEM: A problem whose solution requires a lucky guess or a view of the problem from a different perspective. Puzzles do not always involve mathematics, and careful analysis of the information is often of little help in finding a correct solution. They show students the importance flexibility plays in solving problems.

   Example:  Place the digits 1-9 in the squares so that the sum of each row and column is 15.

*These types of problems are discussed in more detail in Charles, R., & Lester, F. (1982). Teaching problem solving: What, why, and how. Palo Alto, CA: Dale Seymour Publications.

What is a Good Problem?

Good problems are characterized by a number of key attributes. Teachers should keep these attributes in mind as they select and create problems for their students. It is important to note that a good problem does not necessarily need to include all of these characteristics.

- The problem should be readily understandable to the student.
- It should not involve new mathematical concepts.
- It should be intrinsically motivating and intellectually stimulating.
- It should lend itself to more than one method of solution.
- It should be somewhat open-ended so that it can be generalized or extended to a variety of situations.

Sources of Good Problems

A variety of excellent problems is readily available to teachers. Many commercial publications, such as those listed in the bibliography, include problems, solutions, and in some cases, teaching suggestions. Teachers who are members of the National Council of Teachers of Mathematics receive a monthly journal that includes articles on all aspects of problem-solving instruction. For example, a recent issue featured a calendar that listed a mathematical fact or problem for each day of the academic year.

One of the best sources of good problems is the classroom itself. Problems that are a natural extension of class discussions are the best, for they contain a built-in motivation factor. Student-created problems are also excellent for this same reason. It is important to note that good problems are not easy to create without a reasonable amount of experience with solving problems, so first attempts at teacher- and student-made problems should be limited to extensions of previous problems. However, even commercially-produced problems can be adapted to try to capture student interests. It's easy to build in student names, class incidents, school events, etc.

TEACHING PROBLEM SOLVING

The instructional process for teaching problem solving at the middle school level should reflect what we know about good teaching in general. It should involve interesting lessons suited to the level of the learner, and it should be based upon information we have about the characteristics of successful problem solvers. The climate for instruction is critical as is the integration of problem solving into the curriculum.

Many good teachers are uncomfortable with teaching problem solving. They are not used to the different instructional techniques, and they are a little insecure about not knowing in advance exactly how the lesson will proceed or what the answer will be. These feelings are perfectly natural and understandable. At the beginning, teachers should move slowly. They should build in success for themselves as well as for their students by approaching simple problems in a relaxed atmosphere.

Problem-Solving Strategies

Students and teachers need to keep in mind that the process, not the answer is the major goal of problem-solving instruction. One way to emphasize process is to teach specific strategies. Problem-solving strategies are general techniques which can be used to help solve a variety of problems. Because these procedures are not problem-specific, they are an important part of an individual's problem-solving "tool kit." They can be used singly or in combination.

Table 2 lists popular strategies. Guess-and-check, for example, is perhaps the most frequently used problem-solving strategy. By becoming more aware of the various strategies and consciously recognizing the types of situations where they are most useful, students can improve their problem-solving abilities.

Characteristics of Good Problem Solvers

Good problem solvers -- those individuals who are successful in their attempts to apply mathematical concepts and skills to new situations -- share a number of traits:

- A desire to solve problems. They enjoy the challenge of problems and get involved with the problems.
- Perseverence. They are not easily discouraged by initial attempts which are not successful.
- Flexibility. They are willing to try a variety of strategies.

Table 2

Problem-Solving Strategies

LOOK FOR A PATTERN - The patterns of mathematics are limitless. Recognizing patterns allows us to generalize solutions to complex problems.

CONSTRUCT A TABLE - Organizing data in a table is a good way of identifying patterns. It also helps to determine whether needed information is missing.

MAKE AN ORGANIZED LIST - Like constructing a table, this strategy is a systematic way of examining the data from a problem. It allows us to account for all possibilities.

GUESS AND CHECK - Estimation is an important skill for implementing this strategy. Based upon all the pertinent relationships of the problem, a reasonable guess can be made, checked against the problem conditions, and then modified if necessary.

DRAW A PICTURE - A picture, diagram, or graph is often the most effective way of visualizing the relationships among the information given in the problem. This is especially true of geometric problems.

USE OBJECTS/ACT IT OUT - Sometimes problem situations can best be visualized by using three-dimensional objects or by actually going through the motions described in the problem. Tactile involvement at the concrete level is often the key to analyzing relationships.

WORK BACKWARDS - Some problems lend themselves to starting with the desired outcome and working back to the beginning. This sometimes helps to focus our attention when other strategies seem too broad.

WRITE AN EQUATION - Once a relationship among the given information has been determined, it is possible to write an equation which translates the problem into an abstract mathematical sentence. Individuals with greater math backgrounds tend to use this strategy as a first step, often ignoring simpler, more effective strategies.

SIMPLIFY THE PROBLEM - Studying a simpler similar problem allows us to examine relationships more easily. Once a pattern is discovered, it can be generalized to the more complex original problem.

CHANGE YOUR POINT OF VIEW - Once a point of view is established and a plan for solution is formulated, it is very difficult to change, particularly when it had led to success at solving similar problems. But when it leads nowhere, a new point of view is needed. Previous notions about the problem need to be discarded and the problem should be redefined in a completely different way.

BRAINSTORM - One of the most effective ways of solving a problem is to enlist the support of others. Problems in the real world are rarely solved by one person alone. Cooperative problem solving can lead to more efficient and creative solutions.

- Willingness to guess. They take risks by posing methods and answers.

- Hold conversations with themselves. They "talk through" problems probing for patterns and relationships.

- Separate critical elements from irrelevant details.

- Spend a greater proportion of their time analyzing problems than solving them.

Characteristics of Good Problem-Solving Instruction

Well-designed instruction may enhance positive problem-solving traits.

- The teacher should be enthusiastic, interested, involved in solving problems, and should model a positive attitude toward problem solving. There is no question that teacher attitudes have a large effect on student attitudes and success.

- Success should be "built in" for teachers and students. A teacher's initial insecurity can be overcome by building in success and allowing plenty of time to begin to feel comfortable with solving problems. First experiences should be brief and familiar to the students. Later problems should build on these early successes in small increments.

- Students should be encouraged to take risks, to volunteer information and ideas, to try to alternate methods, and to accept the frustrations that come from not knowing. The climate should be "safe" for all students, free from ridicule and with positive reinforcement for risk-taking. Teachers need to be open and non-judgmental in accepting student responses. All suggestions should be valued and used if possible. This is an area where a teacher's lack of experience with problem solving can be an asset. The teacher can very effectively model the risk-taking and cooperative brainstorming which are important for problem-solving instruction.

- Provide time for thinking. Many students have a tendency to stop thinking if a solution isn't immediately evident. Some will stop as soon as another student comes up with a solution. Teachers should provide plenty of time for solving problems. Not every problem can or should be solved in a single lesson; some may take several days to explore completely.

- Emphasis should be placed on the process of solving problems, not just on getting the correct answer. Shifting the emphasis away from the answer is very difficult because of the conditioning we've built into our traditional instruction. The "one-right-answer" approach is actually counterproductive to developing flexibility and creativity in problem solving; eliminating students' preoccupation with the answer is very important. Problems which require analysis and set-up without actually determining a solution can be helpful.

- Students should be allowed to work together. Small group learning is one of the most effective ways of providing a key element of problem solving--interaction. Work in small groups reduces anxiety, encourages cooperation and sharing, and is easier to manage than whole-class teaching. Listening in on student discussions is one of the best ways of seeing the problem-solving process in action.

Instructional Planning

Providing meaningful instruction in problem solving involves a commitment to establishing problem solving as a significant part of the mathematics curriculum. Instruction in this area must not be seen as something "extra" but rather as an important, integral part of the day. Unfortunately, textbooks, in general, do not support this philosophy. As texts are revised, they tend to reflect a greater commitment to problem solving, but they are far from establishing problem solving as a program focus. For this reason, individual teachers will have to take the majority of the responsibility for providing effective problem-solving instruction.

Plan to include a problem-solving experience in each lesson. Students should participate in daily problem-solving experiences. This does not mean that all students should solve a problem everyday. It is possible to provide daily problem- solving instruction with only one problem per week by allocating some time during each lesson to discussion of the problem.

The number of problems presented in a given time period is not the critical factor. The key is what works best for the teacher and the students. As long as daily systematic problem-solving instruction is provided, students will receive the variety of experiences necessary to become successful problem solvers.

The Role of Calculators

Computation is an important part of the process of solving many problems. Because of this, problem solving is an effective way of reinforcing previously-learned arithmetic skills. However, since the role played by computation is only a small part of the total problem-solving process, it is often advisable to allow students to use calculators to assist them with the computation. This will free them from the time-consuming pencil-and-paper calculations and give them more time to concentrate on the other aspects of the problem-solving process.

The calculator also gives students an opportunity to solve problems that would otherwise have been impossible because of tedious computations. Problem-solving strategies such as guess-and-check are effective with complex problems if students are allowed to use calculators.

Successful Instructional Techniques

The following ideas are based upon the experiences of middle school mathematics teachers.

BEGINNING OF THE YEAR

- Do "warm up" problems with the whole class.
- Encourage divergent thinking and cooperation.
- Keep problems brief and build in success.
- Present fewer problems but go over them thoroughly.
- Spend the first month or two working hard on establishing a positive climate for problem solving.
- Model desired problem-solving behaviors: enthusiasm, perseverence, flexibility, etc.

MOTIVATION - A KEY FACTOR FOR MIDDLE SCHOOL STUDENTS

- Create an atmosphere of success.
- Involve students in the problems. Personalize problems for the class. Include student names, current events, sports, etc.
- Have students create their own problems.
- Encourage students to explain their solutions to the class.

- Create problem-solving bulletin boards.
- Post a "Problem of the Week."
- Display student solutions to problems.
- Keep a chart of points earned for problem-solving activities.
- Include questions related to problem solving on tests.

ALLOCATE TIME FOR ANALYZING PROBLEMS

- Have students restate problems in their own words.
- Have students read problems to each other with inflection to try to convey the meaning.
- Have students explain the problems to each other.
- Ask relevant leading questions.
- Present problems with too much or too little information.
- Read problems aloud to the class and ask students to visualize relationships described in the problems.

PROBLEM-SOLVING STRATEGIES

- Accept and value a variety of strategies for solving a problem. Encourage divergent thinking.
- Keep a chart of strategies used. Refer to strategies by name.
- Review previously-learned strategies before presenting a new problem.
- Give hints or clues that can be used or ignored by students as they plan a solution.
- Present problems which involve a variety of different strategies.
- Have students use manipulative materials, pictures, diagrams, charts, and graphs.

Evaluating Student Work

If our goals in problem-solving instruction are to encourage students to take risks, to persevere, to concentrate on process, then it makes little sense to grade answers to problems. Any form of evaluation which focuses on the answer to a problem defeats the purpose of the instruction. Evaluation of student work should mirror instruction. That is, it should be process-oriented rather than outcome-oriented.

Assessing student performance is not synonymous with grading. Assigning letter grades to student work may well detract from attitudinal goals. It is possible to assess student work through careful observation, questioning of students, and analysis of written work.

Point systems can be used to evaluate written work which place the emphasis on understanding and setting up the problem for solution. No matter which system is used, it is important that the students see some value placed on their work. This is necessary for problem solving to be perceived as an important part of the total mathematics program.

FOOTNOTES

[1] National Assessment of Educational Progress. (1983). The third national mathematics assessment: Results, trends, and issues. (Report No. 13-MA-01). Denver: Author.

[2] California Assessment Program. (1983). Student achievement in California Schools. (1982-83 Annual Report). Sacramento: California State Department of Education.

[3] Crosswhite, F. J., Dossey, J. A., Swafford, J. O., McKnight, C. C., & Cooney, T. J. (1985). The second international mathematics summary report for the United States. Champaign, IL: Stipes Publishing Company.

BIBLIOGRAPHY OF SELECTED PROBLEM-SOLVING MATERIALS

BACKGROUND INFORMATION

Charles, R., & Lester, F. (1982). Teaching problem solving: What, why, and how. Palo Alto, CA: Dale Seymour Publications.

Ideas on how to organize, implement, and evaluate a problem-solving program.

National Council of Teachers of Mathematics. (1980). Problem solving in school mathematics. Reston, VA: Author.

Twenty-two essays written by 41 problem-solving experts dealing with a variety of problem-solving topics.

Poyla, G. (1973). How to solve it. Princeton, NJ: Princeton University Press.

This classic work describes a general method for solving problems.

PROBLEMS WITH TEACHING SUGGESTIONS

Charles, R., Mason, R., Garner, D., Nofsinger, J., Moffatt, E., & White, C. (1985). Problem-solving experiences in mathematics. Reading, MA: Addison Wesley Publishing.

An integrated problem-solving instructional program which contains a problem for each day of the school year and extensive teaching guidelines.

Dolan, D., & Williamson, J. (1983). Teaching problem-solving strategies. Reading, MA: Addison Wesley Publishing Company.

Teaching hints and ideas for over 20 problem-solving lessons and follow-up activities which develop six strategies for solving problems.

Greenes, C., Gregory, J., & Seymour, D. (1977). Successful problem-solving techniques. Mountain View, CA: Creative Publications.

Problem-solving techniques and skills are outlined and a variety of problems are presented in a reproducible format.

Krulik, S., & Rudnick, J. (1980). Problem solving: A handbook for teachers. Rockleigh, NJ: Allyn and Bacon.

This handbook discusses important issues in teaching problem solving and presents annotated strategy games and non-routine problems.

Krulik, S., & Rudnick, S. (1984). A sourcebook for teaching problem solving. Rockleigh, MA: Allyn and Bacon.

Teaching suggestions, problems with discussion questions, and over 150 reproducible worksheets.

Lane County Mathematics Project. (1983). Problem solving in mathematics. Palo Alto, CA: Dale Seymour Publications.

Background information and lessons which build problem-solving skills with drill and practice, grade level topics, and challenge activities. For grades 4-9.

Meyer, C., & Sallee, J. (1983). Make it simpler. Reading, MA: Addison Wesley.

Over 100 problem sheets along with teaching suggestions and activities for implementing small-group learning.

Ohio Department of Education. (1980). Problem solving: A basic mathematics goal. Columbus, OH: Author.

Two books which give teaching background information and a good presentation of ways to teach problem-solving strategies.

Overholt, J. L., Rincon, J. B., & Ryan, C. A. (1984). Math problem solving for grades 4 through 8. Rockleigh, NJ: Allyn and Bacon.

Thirty annotated problem-solving lessons with over 100 problems illustrating specific techniques.

Souviney, R. Solving problems kids care about. (1981). Glenview, IL: Scott, Foresman & Company.

A collection of non-routine and applied problems with teaching suggestions and 34 reproducible problem sheets.

PROBLEM SOURCES

Fisher, L. (1982). Super problems. Palo Alto, CA: Dale Seymour Publications.

Forty-eight reproducible problems with solutions and teaching ideas. Available in poster or book form for grades 7-9.

Fisher, L., & Medigovich, W. (1981). Problem of the week. Palo Alto, CA: Dale Seymour Publications.

Ninety reproducible problems with solutions and teaching ideas. Available in poster or book form for grades 9-12.

Greenes, C., Immerzeel, G., Schulman, L., Spungin, R., & Ockenga, E. (1980). Techniques of problem solving. Palo Alto, CA: Dale Seymour Publications.

Developmental workbooks, skill sheets, transparency masters, and card decks which provide sequential instruction in problem solving.

Seymour, D. (1982). Favorite problems. Palo Alto, CA: Dale Seymour Publications.

Forty-eight reproducible problems with solutions and teaching ideas. Available in poster or book form for grades 5-7.

Seymour, D. (1984). Problem parade. Palo Alto, CA: Dale Seymour Publications.

Forty-eight reproducible problems with solutions and teaching ideas. Available in poster or book form for grades 4-6.